The razor applied at the same distance from the fixed end would sometimes cut through the hair before it had bent it as much as 30°; and this shows that a force of half a grain must make the pressure per unit area at the place of contact sufficient to cause crushing or disruption of the material even when the edge has entered the hair to a distance comparable with the radius of the latter.

If we assume that the thickness of the edge is 1/100,000 in. and that it has entered the hair until the length of the edge engaged is 1/1,000 in., the area in contact will be about 1/100,000,000 of a square inch and the pressure per square inch rather more than 3 tons, if the total force over the area of contact is half a grain.

It is difficult to get any direct measure of the pressure required to destroy by crushing or shearing the material of which hair is composed, but horn which is of the same nature requires a much larger pressure than 3 tons per square inch to crush it.

A rough experiment showed that a cylindrical steel punch with a flat end, began to sink into a block of horn when the pressure was between 12 and 16 tons per square inch.

It would seem, therefore, that although the optical method shows that the thickness at the edge cannot be greater than 1/100,000 inch, the real thickness judged by the pressure per unit area necessary to cause the edge to cut in the way it actually does, must be considerably less than this.

"On the Determination of the Wave-length of Electric Radiation by Diffraction Grating." By JAGADIS CHUNDER BOSE, M.A. (Cantab.), D.Sc. (Lond.), Professor of Physical Science, Presidency College, Calcutta. Communicated by LORD RAYLEIGH, Sec. R.S. Received June 2,—Read June 18, 1896.

While engaged in the determination of the "Indices of Refraction of various Substances for the Electric Ray" (*vide* 'Proceedings of the Royal Society,' vol. 59, p. 160), it seemed to me that the results obtained would be rendered more definite if the wave-length of the radiation could at the same time be specified. Assuming the relation between the dielectric constant K and the index μ as indicated by Maxwell, to hold good in all cases, it would follow that the index could be deduced from the dielectric constant and *vice versâ*. The values of K found for the same substance by different observers are, however, found not to agree very well with each other. This may, to a certain extent, be due to the different rates of alternation of the field to which the dielectrics were subjected. It has been found in general that the value of K is higher for slower rates of alternation

and the deduced value of μ would therefore be higher for slow oscillations, the longer waves being thus the more refrangible. The order of refrangibilities would in such a case appear to be somewhat analogous to that in an anomalously dispersive medium like iodine vapour.

With exceedingly quick ethereal vibrations which give rise to light, there is an inversion of the above state of things, *i.e.*, the shorter waves are generally found to be the more refrangible. It would thus appear that there is a neutral vibration region for each substance at which this inversion takes place, and where a transparent medium produces no dispersion.

It would be interesting to be able to determine the indices of refraction corresponding to different wave lengths, chosen as widely apart as possible, and plot a curve of refrangibilities. A curve could thus be obtained for rock salt, which is very transparent to luminous and obscure radiations, and fairly so to electric radiation. Carbon bisulphide, which is very transparent to all but the ultra-violet radiation, would also be a good substance for experiment.

For the construction of a curve of refrangibility for electric rays, having different vibration frequencies, the indices could be determined by the method of total reflection referred to above. The determination of the corresponding wave-lengths, however, offers great difficulties. Hertz used for this purpose the method of interference, the positions of nodes and loops of stationary undulation produced by perpendicular reflection being determined by means of tuned circular resonators.

Sarasin and De la Rive subsequently repeated these experiments with different sized vibrators and resonators. They found that the apparent wave-length depended solely on the size of the resonators. The wave-length found was approximately equal to eight times the diameter of the circular resonator. From these experiments it was supposed that the radiator emitted a continuous spectrum consisting of waves of different lengths, and that the different receivers simply resonated to vibrations with which they happened to be in tune. If this supposition be true the emitted radiation should, by the action of a prism, or better still, a diffraction grating, spread out in the form of a continuous spectrum. If, on the contrary, the radiation is monochromatic, the spectrum should be linear. The experiments to be described below may throw some light on this question.

Professor J. J. Thomson, referring to the above case, is of opinion that the hypothesis of a continuous spectrum is highly improbable. It is more likely that, owing to the oscillation being of a dead-beat character, the resonator is set in vibration by the impact of incident electric waves. Each resonator vibrating at its particular free period,

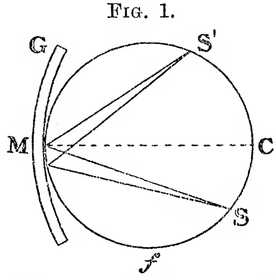
measures its own wave-length. There is, however, one difficulty in reconciling the theoretical value with that actually obtained. According to theory, the wave-length should be equal to twice the circumference, or 2π times the diameter of the circular resonator. The value actually obtained by Messrs. Sarasin and De la Rive is, as has been said before, eight times the diameter of the circle.

Rubens, using a bolometer and Lecher's modification of the slide bridge, determined the nodes and loops in a secondary circuit in which stationary electric waves were produced. A curve obtained by representing the bolometer deflections as ordinates and the distances of the bridge from one end as abscissæ, shows the harmonic character of the electric disturbance in the wire. It was found that the wave-length obtained by this method did not depend on the period of the primary vibrator; the wave-length measured was merely that of the free vibration started *in the secondary circuit* by the primary disturbance.

Hertz's method is therefore the only one for the measurement of electric waves in air, and the result obtained by this method is vitiated by the influence of the periodicity of the resonator. It was therefore thought desirable to obtain the wave-length of electric radiation in free space by a method unaffected by any peculiarity of the receiver.

I have succeeded in determining the wave-length of electric radiation by the use of curved gratings, and the results obtained seem to be possessed of considerable degrees of accuracy. Rowland's method of using the curved grating for obtaining diffraction light spectra was also found well suited for the production of pure spectra of electric radiation. The focal curve f in this arrangement is a circle, having as a diameter the straight line joining the centre of curvature C with the apex M of the grating.

FIG. 1.



G, the grating; M, its apex; f, the focal curve.

A source of radiation situated on this curve will give a diffracted spectrum, situated on the same curve defined by the equation

$$(a+b)(\sin i \pm \sin \theta) = n\lambda,$$

where $a+b$ is the sum of breadths of strip and space in the grating, i = angle of incidence, θ = angle of diffraction. The sign of θ is taken positive when it lies on the same side of the normal as the incident radiation.

In the above equation there are two interesting cases :—

(1) When the receiver is placed at C, $\theta = 0°$

$$(a+b) \sin i = n\lambda.$$

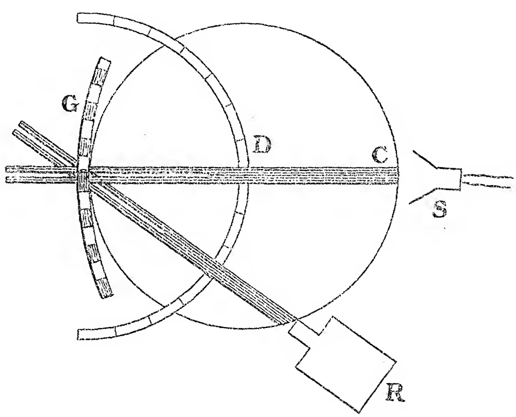
(2) When the deviation is minimum $i = \theta$

$$2(a+b) \sin i = n\lambda.$$

Arrangement of the Apparatus.

The grating, which is cylindrical, is placed vertically on a wooden table, with its centre at C, occupied in the diagram by the spiral spring coherer S. With the radius, which joins the centre to the apex of the grating, as a diameter, a circle is engraved on the table—the focal curve—on which the radiator and the receiver are always kept. A pin is fixed immediately below the apex, and a graduated ring sunk in the table with this pin as the centre. The graduated

FIG. 2.



The radiator, R, and the receiver, S, revolve round a pivot vertically below the apex of the grating, along the focal curve. The angles are measured by the graduated circle, D.

circle is used for the measurement of the angles of incidence and diffraction. Two radial arms revolving round the pin carry the radiator and the receiver. The ends of the arms near the pin have

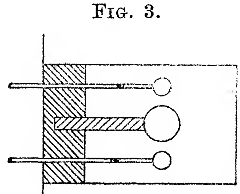
narrow slits, through which the pin projects. The slits allow the necessary sliding for placing the radiator and the receiver on the focal curve. It would be better to have the sliding arrangement at the free ends of the arms, the pin passing through the central ends, acting as a pivot. The circle is graduated into degrees, but one-fourth of a degree may be estimated.

Description of the Apparatus.

The Radiator.—Electric oscillation is produced between two metallic beads and an interposed sphere 0·78 cm. in diameter. The beads and the interposed sphere were at first thickly coated with gold, and the surface highly polished. This worked satisfactorily for a time, but, after long-continued action, the surface of the ball became roughened, and the discharge ceased to be oscillatory. After some difficulty in obtaining the requisite high temperature, I succeeded in casting a solid ball and two beads of platinum. There is now no difficulty in obtaining an oscillatory discharge, and the ball does not require so much looking after.

As an electric generator, I at first used a small Ruhmkorff's coil, actuated by a battery. I, however, soon found that the usual vibrating arrangement is a source of trouble; the contact points soon get worn out, and the break becomes irregular. The oscillation produced by a single break is quite sufficient for a single experiment, and it is a mere waste to have a series of useless oscillations. But the most serious objection to the continuous production of secondary sparks is the deteriorating action on the spark balls. Anyone who has tried to obtain an oscillatory discharge knows how easily the discharge becomes irregular, and the most fruitful source of trouble is often traced to the disintegration of the sparking surface. In my later apparatus I have discarded the use of the vibrating interrupter. The coil has also been somewhat modified. A long strip of paraffined paper is taken, and tinfoil pasted on opposite sides; this long roll is wound round the secondary to act as a condenser, and appropriate connexions made with the interrupting key. This arrangement

Fig. 3.

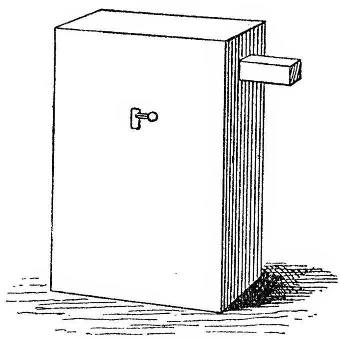


The Radiator.

secures a great saving of space. Two jointed electrodes carry the two beads at their ends; the distance between the beads and the interposed ball can be thus adjusted. This is a matter of importance, as the receiver does not properly respond when the spark-length is too large. Small sparks are found more effective with the receiver used. After a little experience it is possible to tell whether the discharge is oscillatory or not. The effective sparks have a smooth sound, whereas non-oscillatory discharges give rise to a peculiar cracked sound, and appear jagged in outline.

The wires of the primary coil are in connexion with a small storage cell through a tapping key. The coil, a small storage cell, and the key are enclosed in a tinned iron box. It must be borne in mind that a magnetic disturbance is produced each time the primary

Fig. 4.



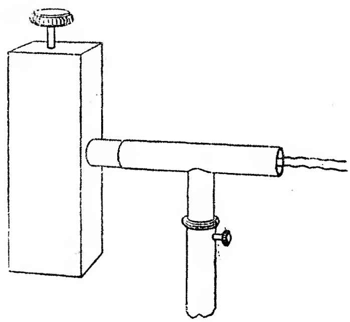
The Radiating Box, one-fifth natural size.

circuit of the induction coil is made or broken; a sudden variation of the magnetic field disturbs the receiver. The iron box in which the coil is enclosed screens the space outside from magnetic disturbance. On one side of the box there is a narrow slit through which the stud of the press-key projects. In front of the box is the radiator tube, which may be square or cylindrical. The radiating apparatus used in the following experiments has a square tube 1 sq. in. in section. The apparatus thus constructed is very portable. The one which I often use is 7 in. in height, 6 in. in length, and 4 in. in breadth. To obtain a flash of radiation it is merely necessary to press the key and then release it. The break is made very sudden by an elastic spring.

The Spiral Spring Receiver.—The receiving circuit consists of a spiral spring coherer in series with a voltaic cell and a dead-beat galvanometer of D'Arsonval type. An account of this form of receiver has already been given (*vide* "On the Indices of Refraction of

various Substances for the Electric Ray," 'Roy. Soc. Proc.,' vol. 59, p. 163). The receiver is made linear by arranging bits of steel spiral springs side by side, the sensitive surface being 3 mm. broad and 2 cm. in length. An electrical current enters along the breadth of the top spiral and leaves by the lowest spiral, having to traverse the intermediate spirals along the numerous points of contact. The resistance of the receiving circuit is thus almost entirely concentrated

FIG. 5.

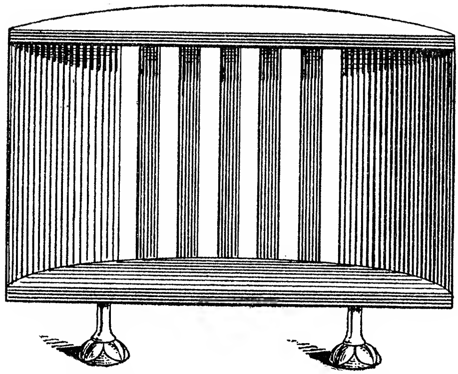


The Spiral Spring Coherer.

at the sensitive contact surface, there being little useless short circuiting by the mass of the conducting layer. When electric radiation is absorbed by the sensitive surface, there is a sudden diminution of the resistance, and the galvanometer in circuit is violently deflected. By adjusting the electromotive force of the circuit the sensitiveness of the receiver may be increased to any extent desirable. The receiver at each particular adjustment responds best to a definite range of vibration lying within about an octave. The same receiver could, however, be made to respond to a different range by an appropriate change of the electromotive force acting on the circuit. Very careful adjustment of the E.M.F. of the circuit is necessary to make the receiver respond at its best to a particular range of electric vibration.

The Cylindrical Grating.—The source of radiation—the spark gap —being a line, the curved diffraction grating is made cylindrical. The spark gap is always kept vertical; the grating is made of equidistant metallic strips, which are vertical and parallel. A piece of thin sheet ebonite is bent in the shape of a portion of a cylinder and kept in that shape by screwing against upper and lower circular guide pieces of wood. Against the concave side of the ebonite are stuck strips of rather thick tinfoil at equal intervals. Five different

Fig. 6.



The Cylindrical Diffraction Grating.

gratings were thus made with strips or spaces equal to 3 cm., 2·5 cm., 2 cm., 1·5 cm., and 1 cm. respectively.

The diameter of the cylindrical grating is 100 cm. It would perhaps have been better to use a grating with a less curvature, but it must be remembered that the intensity of radiation is very feeble, and I was apprehensive of the receiver failing to respond when placed at too great a distance. I find from the sensibility of the receiver used that it would be possible to increase the diameter of the cylinder to about 150 cm., and this size I intend to use in the construction of my next grating. The aperture of the grating is in the following experiments reduced to the smallest practicable limit.

Account of the Experiments.

The receiver being placed at a suitable position on the focal curve, the radiator is moved about on the same curve till the diffracted image falling on the receiver produces response in the galvanometer. The procedure adopted is as follows. The receiver is placed, say, at the centre of the grating ($\theta = 0°$). The electric ray at first falls on the grating at a large angle of incidence. A series of flashes of electric radiation are now produced by manipulating the key, and the angle of incidence gradually decreased till the receiver suddenly responds. The angle of incidence corresponding to the zero angle of diffraction is thus determined. The receiver is then placed at a new position on the focal curve, and the corresponding angle of incidence determined as before. In this way a series of angles of incidence, with their corresponding angles of diffraction, are found for each grating.

It should be remarked here that numerous difficulties were encountered in carrying out the experiments. The reflections from the walls of the room, from the table, &c., were at first sources of considerable trouble. By taking special care, I succeeded in eliminating these disturbances. The radiating balls were placed about 1 cm. inside the square tube. This prevented the lateral waves acting on the receiver. The receiver was provided with a guard tube, which stopped all but the diffracted radiation reaching the sensitive surface. The insulated wires from the ends of the receiver were protected by thick coatings of tinfoil, and led to the galvanometer, which was placed at a considerable distance. The cell and the galvanometer were enclosed in a metallic case with a narrow slit for the passage of light reflected from the galvanometer.

In spite of all these precautions, I was baffled for more than six months by some unknown cause of disturbance which I could not for a long time account for. It was only recently, when nearly convinced of the futility of further perseverance, that I discovered the mistake in supposing sheets of tinned iron to be perfectly opaque to electric radiation. The metal box which contains the radiating apparatus seems to transmit a small amount of radiation through its walls, and if the receiver happens to be in a very sensitive condition it responds to the feeble transmitted radiation. I then made a second metallic cover for the radiating box, which precaution was found effective, provided the receiver was not brought very close to the radiator. The receiver is still affected if placed *immediately above* the radiator tube, though two metallic sheets be intervening. For this reason I had to postpone taking the reading for minimum deviation till I had made a radiation-proof box. A soft iron box (to prevent escape of magnetic lines of induction), enclosed in a second enclosure of thick copper, would, I expect, be found impervious to electric radiation.

With the second protective enclosure, all difficulties were practically removed. As a test for the absence of all disturbing causes, I observed whether the receiver remained unaffected when the grating was "off." There is a further test for the absence of external disturbances. The response, if only due to the diffracted beam, depends on the position of the radiator on the focal curve. If this angle of incidence is decreased, there should then be no action on the receiver. I found the positions of the radiator on the focal curve producing action on the receiver, to be well defined, and I experienced no further disturbance due to stray radiations.

The grating is fixed vertically on the table, so that its centre is at the same height as that of the middle of the receiving and radiating tubes. A small mirror is fixed at the middle of the central strip. The observer, placing his eye at the same height as that of the

radiator, levels the grating till the image of the eye is seen reflected by the mirror.

I first obtained an approximate value of the wave-length with a 2-cm. grating, and then took careful and systematic readings with the different gratings. By different gratings is meant the same curved piece of ebonite, on which strips of different breadths were successively applied. The grating was found fairly adjusted, and the readings taken on the right side of the grating agreed well with the corresponding ones on the left side. I did not, therefore, think it necessary to take double readings, but took the various readings alternately on the right and on the left side. In one case only I found the grating on one side giving slightly better reading than the other. When the incident angle is too oblique, the diffracted image is not sharp, and I therefore did not extend the reading beyond 40° of incidence. Spectra of the first order only were observed. The response in the receiving circuit was somewhat feeble when 1 cm. or 1·5 cm. grating was used. But a 2-cm. grating gave stronger indications. With 2·5 and 3 cm. gratings the response was very energetic and the definition of the diffracted spectrum very sharp. For example, when the receiver was kept fixed, and the angle of incidence gradually varied, there was an abrupt and strong response produced in the receiving circuit, as soon as the angle of incidence attained the proper value. A slight variation of this angle, even of less than a quarter of a degree, produced displacement of the diffracted image, and there was then no further action on the receiver. Had my graduated circle permitted it, I could have got more accurate readings. The radial arms carrying the receiver and radiator were of too primitive a design to make it worth while to attempt greater accuracy. I give below the readings of the angles of incidence and the corresponding angles of diffraction obtained with the different gratings, and the wave-length deduced from them.

Grating A.—Breadth of strip = 1 cm.

i.	θ.	λ.	Mean λ for A.
38·0°	18°	1·849	
35·0	20	1·831	1·843
37·0	19	1·854	
38·75	17	1·837	

Grating B.—Breadth of strip = 1·5 cm.

i.	θ.	λ.	Mean for B.
38·0°	0°	1·847	
26·0	10	1·836	1·844
28·5	8	1·849	

Grating C.—Breadth of strip = 2 cm.

i.	θ.	λ.	Mean for C.
27·5°	0°	1·846	
22·0	5	1·847	1·849
20·0	7	1·855	

Grating D.—Breadth of strip = 2·5 cm.

i.	θ.	λ.	Mean for D.
21·5°	0°	1·832	
29·5	− 7	1·852	1·845
33·0	−10	1·854	
34·0	−11	1·841	

Grating E.—Breadth of strip = 3 cm.

i.	θ.	λ.	Mean for E.
18·0°	0°	1·854	
23·25	− 5	1·845	1·848
25·5	− 7	1·851	
31·0	−12	1·843	

It would thus be seen that the different values of wave-length obtained from the above experiments are concordant, the mean value being 1·846 cm.

I then carefully removed the electrical vibrator, and measured approximately the size of the sparking balls. The radiator, it must be remembered, was placed vertically inside a square tube, each of whose sides is 2·5 cm. The radiator was about 1 cm. inside from the free end of the tube.

The diameter of the central ball = 0·78 cm.
„ each side bead = 0·3 „
Distance between the outer surfaces of the beads = 1·5 cm.
„ „ inner (sparking) surfaces „ = 0·9 „

The wave-length, 1·84, is almost exactly equal to twice the distance between the sparking surfaces of the beads. Without further experiments with different sized radiators, it is difficult to say whether the above simple relation is accidental or not. The following rough determinations, made with a second radiator, may be of some interest in connexion with the above. I took off the central sphere from the radiator used in the last experiment, and substituted a larger ball. The distance between the inner sparking surfaces is then 1·2 cm.

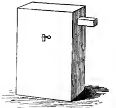
Breadth of Strip = 3 cm.

i.	θ.	λ.	Mean.
23·0°	0°	2·34	
29·0	−5	2·38	2·36
34·5	−10	2·36	

The wave-length found is approximately equal to 2·36 cm., and twice the distance between the sparking surfaces is 2·40 cm.

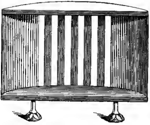
Conclusion.—The experiments described above seem to prove that the diffracted spectrum is not continuous, but linear. The method of determining the wave-length of electric radiation by diffraction grating is seen to give results which are concordant. The determinations are not affected by the periodicity of the receiving circuit, the receiver being simply used as a radioscope. With a better mounting and a finely graduated circle, it would be possible to obtain results with a far greater degree of accuracy. I hope to send, in a future communication, the results obtained with a better form of apparatus, with which I intend to study the relation of the wave-length with the size of the radiator, and the influence of the enclosing tube on the wave-length. I shall at the same time send an account of transmission gratings.

FIG. 4.



The Radiating Box, one-fifth natural size.

Fig. 6.



The Cylindrical Diffraction Grating.